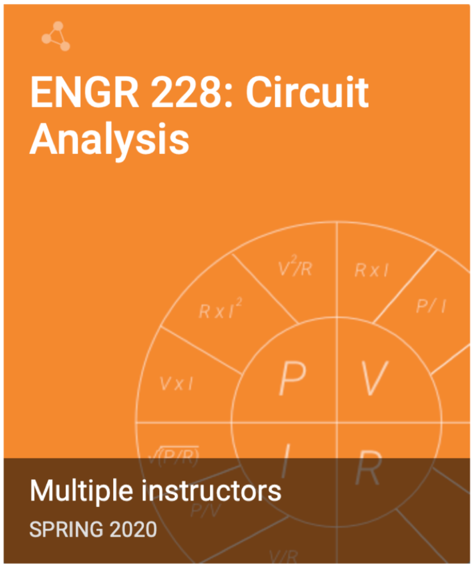

Chapter 8.1-2

Periodic Waveforms and Average Power

Engr228 - Circuit Analysis

Spring 2020

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Sections 8.1-2 Objectives

- Understand the following sinusoidal steady-state power concepts:
  - Instantaneous power;
  - Average power;
  - Root Mean Squared (RMS) and effective values.

Instantaneous Power

- **Instantaneous power** is the power measured at any given instant in time.
- In DC circuits, power is measured in watts as:

$$P = vi = i^2 R = \frac{v^2}{R}$$

- In AC circuits, voltage and current are time-varying, so instantaneous power is time-varying. Power is still measured in watts as:

$$P = vi = i^2 R = \frac{v^2}{R}$$

Instantaneous Sinusoidal Steady-State Power

$$\boxed{\mathrm{p(t) = v(t) \times i(t)}}$$

- In an AC circuit, voltage and current are expressed in general form as:

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

- Instantaneous power is then:

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

Instantaneous Power - Continued

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

- Using the trig identities:

$$\cos(x)\cos(y) = 0.5\{\cos(x-y) + \cos(x+y)\}$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(2\omega t + \theta_V - \theta_I) = \cos(\theta_V - \theta_I)\cos(2\omega t) - \sin(\theta_V - \theta_I)\sin(2\omega t)$$

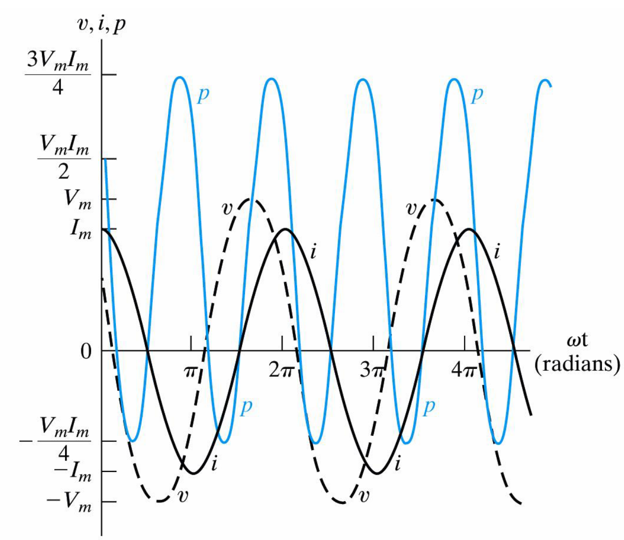
it can be shown that instantaneous power is:

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- Note that the instantaneous power contains a constant term as well as a component that varies with time at *twice the input frequency.*

Instantaneous Power Plot

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$



Instantaneous vs. Average Power

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

The equation above can be simplified as follows:

$$p(t) = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

where

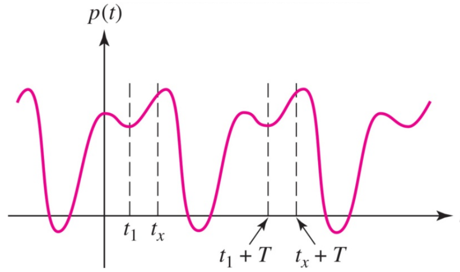
$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) \qquad \text{(Average Power)}$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) \qquad \text{(Reactive Power)}$$

Instantaneous vs. Average Power

- **Average power** is sometimes called *real power* because it describes the power in a circuit that is transformed from electric to nonelectric energy, such as heat. Average power is also the average value of the instantaneous power over one period:

$$P = \frac{1}{T}\int_{t_x}^{t_x+T} p(t)\,dt$$



Average Power - Continued

$$P = \frac{1}{T}\int_{t_x}^{t_x+T} p(t)\,dt$$

$$p(t)= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

The last two terms in the equation above will integrate to zero since the average value of sin and cosine signals over one period is zero. Therefore:

$$P_{AVG} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

Example - Power Calculations

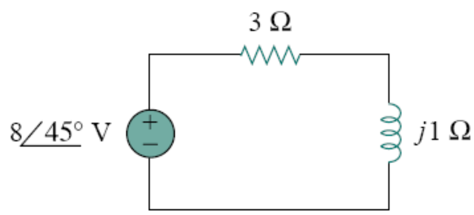
Calculate the instantaneous and average power if:

v(t) = 80cos(10t + 20º)V
i(t) = 15cos(10t + 60º)A

$P_{inst}(t) = 459.6 + 600\cos(20t + 80^o)W$
$P_{avg} = 459.6W$

Example - Average Power

Calculate the average power absorbed by the resistor and inductor and find the average power supplied by the voltage source.



P_R = 9.6W absorbed

P_L = 0W

P_{Source} = 9.6W delivered

The Power Scene for Resistors

$$p(t)=\frac{V_mI_m}{2}\cos(\theta_v-\theta_i)+\frac{V_mI_m}{2}\cos(\theta_v-\theta_i)\cos(2\omega t)-\frac{V_mI_m}{2}\sin(\theta_v-\theta_i)\sin(2\omega t)$$

- Since the voltage and current are in phase across a resistor, the instantaneous power becomes:

$$p(t) = P + P\cos(2\omega t)$$

- Integrating over one period shows again that average power in a resistor is:

$$P = \frac{V_mI_m}{2}$$

- Note that average power in a resistor is *always positive* meaning that power cannot be extracted from a purely resistive network.

The Power Scene for Inductors

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- For inductors, $\theta_v - \theta_i$ = +90º or the current lags the voltage by 90º. The instantaneous power then reduces to:

$$p = -Q\sin 2\omega t$$

- Note that Q is *always* a positive quantity for inductors. The average power absorbed by an inductor is zero, meaning energy is not transformed from electric to non-electric form. Instead, the instantaneous power is continually exchanged between the circuit and the source at a frequency of 2ω. When p is positive, energy is stored in the magnetic field of the inductor. When p is negative, energy is transferred back to the source.

The Power Scene for Capacitors

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- For capacitors, $\theta_v - \theta_i = -90$º or the current leads the voltage by 90º. The instantaneous power then reduces to (just like inductors):

$$p = -Q\sin 2\omega t$$

- Note that Q is *always* a negative quantity for capacitors. The average power absorbed by a capacitor is also zero, meaning energy is not transformed from electric to non-electric form. Instead, the instantaneous power is continually exchanged between the circuit and the source at a frequency of 2ω. When p is positive, energy is stored in the electric field of the capacitor.

Power - Units

- Power in resistors is called *average*, or *real* power. Power in capacitors and inductors is called *reactive* power, recognizing that their impedances are purely reactive. To distinguish between these different types of power, we use different units – average power is measured in *watts* (W) and reactive power is measured in *volt-amp-reactive* (VAR).

Example 10.1 – Nilsson 11^{th}

1. Calculate the average and reactive power if:

   $v(t) = 100\cos(\omega t + 15^{o})$V

   $i(t) = 4\sin(\omega t - 15^{o})$A

2. State whether the circuit is absorbing or delivering average and reactive powers.

   $P_{avg} = -100W$

   $Q_{reac} = 173.21VAR$

   Delivering average power and absorbing reactive (magnetizing vars) power

Effective or RMS Values

- Sometimes using average AC power values can be confusing. For instance, the average DC power absorbed by a resistor is $P = V_M I_M$ while the average AC power is $P = V_M I_M/2$. By introducing a new quantity called ***effective value****,* the formulas for the average power absorbed by a resistor can be made the same for dc, sinusoidal, or any general periodic waveform.
- The ***effective value*** of a periodic voltage is the *DC voltage that delivers the same average power to a resistor as the periodic AC voltage*.

Effective or RMS Value

For any periodic function x(t), the effective or ***root mean squared (rms)*** value, is given by:

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}$$

Effective or RMS Value

- For example, the effective value of $I = I_m \cos\omega t$ is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t dt}$$

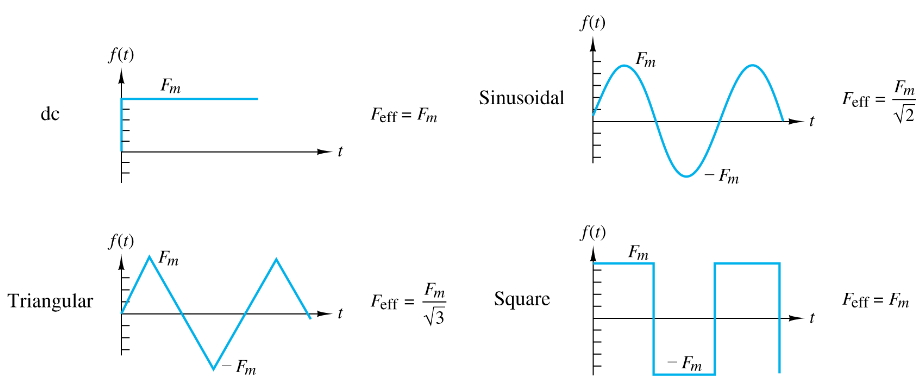
$$I_{rms} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)dt} = \frac{I_m}{\sqrt{2}}$$

- Average power can then be written in two ways:

$$P_{AVG} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$P_{AVG} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

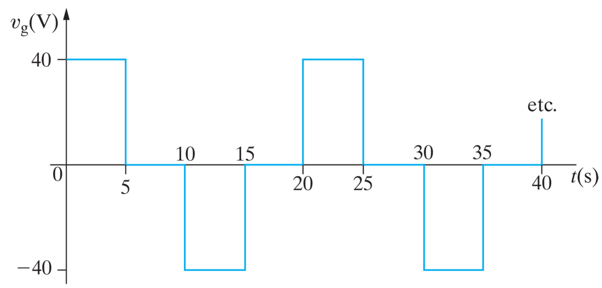
Effective Values of Common Waveforms



Textbook Problem 10.13 - Nilsson 11th

a) Find the rms value of the periodic voltage shown in the figure below.

b) If the voltage is applied across a 40Ω resistor, find the average power dissipated by the resistor.



$V_{rms} = 28.28 V_{RMS}$

$P = 20W$

The Angle $\theta_V - \theta_i$

- $\theta_V - \theta_i$ is known as the ***Power Factor Angle.***
- $\cos(\theta_V - \theta_i)$ is known as the ***Power Factor (PF)***
  - For a purely resistive load, $PF=1$;
  - For a purely reactive load, $PF=0$;
  - For any practical circuit, $0 \le PF \le 1$.
- We must distinguish between positive and negative arguments for the power factor since cos(x) = cos(-x).

Sections 8.1-2 Summary

From the study of this section, you should understand the following sinusoidal steady-state power concepts:

- Instantaneous power;
- Average power;
- Root Mean Squared (RMS) and effective values.